LETTER

Optimal Positioning Scheme of Multiple UAVs through DOP Minimization for Location Identification of Unknown Radar

Jisoo KIM[†], **Seonjoo CHOI**[††], *and* **Jaesung LIM**[†a)], *Nonmembers*

SUMMARY In time difference of arrival-based signal source location estimation, geometrical errors are caused by the location of multiple unmanned aerial vehicles (UAV). Herein, we propose a divide-and-conquer algorithm to determine the optimal location for each UAV. Simulations results confirm that multiple UAVs shifted to an optimal position and the location accuracy improved.
key words: *DOP, TDOA, UAV, relay, location estimation*

1. Introduction

In modern battlefields, the scope of unmanned aerial vehicle (UAV) applications in the field of defense is expanding considerably. In particular, the technology for detecting unknown signals using UAV and for estimating the location of a signal source from a distance through a relay improves network survivability and efficiency by reducing expensive systems and manpower [1].

When estimating the location of an unknown radar signal source using multiple UAVs, the radar signal received by each UAV features a different pulse-arrival time. Therefore, to identify a time difference of arrival (TDOA)-based location, a time difference must be derived [2]. In addition, The Dilution of Precision (DOP) of UAVs should be reduced because geometrical errors that affect the location estimation accuracy occur depending on the location of each UAV [3]. Therefore, an algorithm that efficiently calculates and shifts the position of each UAV is necessitated to minimize the DOP in a three-dimensional (3D) environment.

Recently, an algorithm for selecting an optimal drone set that minimizes the DOP among stationary swarm drones has been proposed to improve location estimation accuracy [4]. However, DOP minimization via the proposed algorithm [4] is limited because the latter selects from a set of deployed drones. Therefore, to respond quickly to an unknown radar, an algorithm that can minimize the estimated location error by rapidly moving the drone set to the optimal location within the operational space of the drone is required.

In this letter, we propose an optimal positioning technique through the maneuvering of UAVs to enable the location identification of unknown radars using multiple UAVs. First, each UAV receives a radar signal and immediately relays it. Subsequently, a ground control station (GCS) estimates the source location based on the TDOA information of the UAV signal. Next, the DOP value is minimized via sequential horizontal and vertical movements of the UAV based on the estimated radar location. At this time, each UAV moves individually to the point with the lowest DOP value among the movable points using the reference node. Simulations confirmed that each UAV efficiently estimated the location of the signal source by shifting the algorithm to the position with the lowest DOP by repeating the algorithm without calculating the entire allowable operating area.

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[†]The authors are with Dept. of Military Digital Convergence, Ajou University, Suwon, South Korea.
[††]The author is with Dept. of AI Convergence Network, Ajou University, Suwon, South Korea.
a) E-mail: jaslim@ajou.ac.kr (Corresponding author)
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2. Related Works

2.1 TDOA

The TDOA does not measure the absolute time, but calculates the time difference of the signals received by each synchronized receiver, thus allowing location estimation without time synchronization between the signal source and receiver [5]. In a 3D environment, the distance d_i between the signal source $P_s = [x_s, y_s, z_s]$ and receiver $P_i = [x_i, y_i, z_i]$ is expressed as

$$d_i = \sqrt{(x_s - x_i)^2 + (y_s - y_i)^2 + (z_s - z_i)^2} \tag{1}$$

$$\Delta d_{i,j} = \hat{d}_i - \hat{d}_j = c(t_i - t_j) \tag{2}$$

In (2), the TDOA can be obtained from the time offset $(t_i - t_j)$ between the i^{th} node, j^{th} node nodes and the speed of light c, and the goal is to obtain the coordinate $\hat{P}_s$ of the signal source that satisfies the TDOA between n nodes.

2.2 DOP

DOP is a measure of position accuracy that expresses the degree of error caused by the geometry between the signal source and the receiving sensor nodes; it is defined as the ratio of the position error to the measurement error. The DOP varies depending on the arrangement of the sensor nodes, and as the DOP decreases, the estimated error decreases [6]. Because d_i is a nonlinear equation, factors that affect the DOP must be derived via linearization. The equation linearized via Taylor series expansion [7] is expressed as

$$\partial D = H \partial S \tag{3}$$

Where ∂D is the variance of d_n, $H = [h_1, \cdots, h_n]^T$ is the Jacobian matrix, and ∂S is the variance of the estimated coordinate error of P_s. The relationship between the covariance matrix cov(∂S) and DOP is derived as follows:

$$\mathrm{cov}(\partial S) = (H^T H)^{-1}\sigma_{UERE}^2 \tag{4}$$

The signal source estimated location error cov(∂S) is calculated as H and σ_{UERE}^2, where σ_{UERE}^2 is the user equivalent range error [8]. The DOP is defined as follows via the calculation of the diagonal element of $(H^T H)^{-1}$.

$$\mathrm{PDOP} = \sqrt{D_{xx} + D_{yy} + D_{zz}},\ \mathrm{HDOP} = \sqrt{D_{xx} + D_{yy}} \tag{5}$$

Here, PDOP/HDOP represents position/horizontal DOP.

3. Optimal Positioning Technique of Multiple UAVs

3.1 System Model

It is assumed that n UAVs within the control range of the GCS hover in the operation area as receiving nodes. The line of sight is guaranteed for each UAV, and location coordinates $P_n = [x_n, y_n, z_n]$ as well as the accurate time synchronization between the GCS and UAV can be obtained via a global positioning system (GPS). The unknown signal source is a ground-fixed air surveillance radar that radiates a narrow beam in a fixed direction with a constant pulse repetition interval (PRI) and rotates mechanically. The UAV relays the signal information received from the unknown radar to the GCS location $P_G = [x_G, y_G, z_G]$. The GCS derives the time difference Δt using the coordinates of each UAV, the time of arrival (TOA) of the received signal, and the PRI of the previously acquired signal source. Subsequently, it estimates the location of the TDOA-based signal source. The GCS repeats the process of estimating the location by moving the UAV to the position where the estimated coordinate-based DOP decreases.

Figure 1 shows a system model of the proposed algorithm. Considering the time offset in a 3D environment, at least four receiving nodes are required to estimate the location of the unknown signal source.

3.2 Proposed Scheme

In this letter, we propose a two-step algorithm for shifting and estimating n UAVs to optimal positions to improve the accuracy of the 3D TDOA estimation. When a signal is received, the GCS derives Δt to estimate its location. Subsequently, a reference node is selected, the HDOP is minimized through the horizontal movement of each UAV, and the PDOP is minimized through the vertical movement.

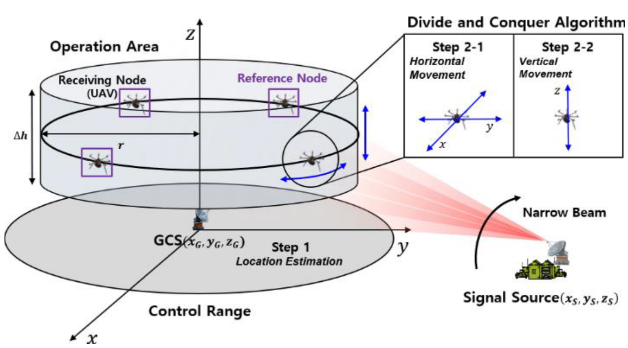

Fig. 1 System model for estimating location of unknown signal source using multiple UAVs

Step 1: Estimate the location of the signal source by deriving the Δt of the signal received by each UAV. Among the n nodes, the signal of the first received node is selected as the reference signal t_0, and Δt is derived using the PRI of the signal source and the TOA(t_i) of the other nodes.

$$\Delta t = t_i - t_0 - k * PRI \tag{6}$$

In general, the PRI of the radar is longer than the propagation delay reaching the receiving node, which allows the reflected signal of the near/far target to be distinguished based on the radiation time [9]. Similarly to (6), Δt can be derived from t_i of the node that receives the radiated signal after k PRIs from t_0. Subsequently, the initial location of the TDOA-based signal source can be estimated using the Δt of the signal received by n nodes. The estimated signal source location is $\widehat{P}_s = [\widehat{X}_s, \widehat{Y}_s, \widehat{Z}_s]$.

Step 2: In this study, the node selection technique proposed in [4] is applied to efficiently identify the locations to which n nodes will shift based on the reference node. Based on n-1 reference nodes, a node is shifted to a point where the DOP is the minimum among the remaining movable range, and the reference node is changed to calculate and shift the other nodes successively using the same method. The movable range refers to the coordinates R_n from which the node can shift horizontally or vertically for a certain duration.

Step 2-1: The UAV is shifted horizontally while considering the $\widehat{P}_s$-based HDOP. At the same initial altitude of each UAV, the node is shifted horizontally by step size Δh to the coordinates that form the minimum HDOP using $\widehat{P}_s$ and the reference node. Subsequently, it is shifted sequentially in the same way for the remaining nodes, and the reference node is changed at every iteration.

$$\min_{P_n \in R_n}(HDOP_{P_n}) = \min_{X_n, Y_n \in R_n}\left(\sqrt{D_{xx} + D_{yy}}\right) \tag{7}$$

Using (7), the point at which P_n forms the minimum HDOP within the movable range R_n can be obtained. In Step 1, the reference node $\{P_n\}_{n=1}^{n-1}$ and altitude $\{z_n\}_{n=1}^{n}$ are fixed and calculated. When the movement of n nodes is completed through the process above, the signal source location is re-estimated and the same process is repeated.

Step 2-2: The UAV is shifted vertically while considering the $\widehat{P}_s$-based PDOP. If Step 2-1 is repeated, then each UAV converges to the position with the lowest HDOP. Subsequently, as in Step 1, the node is shifted vertically by step size Δv to the coordinates that form the minimum PDOP using $\widehat{P}_s$ and the reference node.

$$\min_{P_n \in R_n}(PDOP_{P_n}) = \min_{X_n, Y_n, Z_n \in R_n}\left(\sqrt{D_{xx} + D_{yy} + D_{zz}}\right) \tag{8}$$

Using (8), the point where P_n forms the minimum PDOP

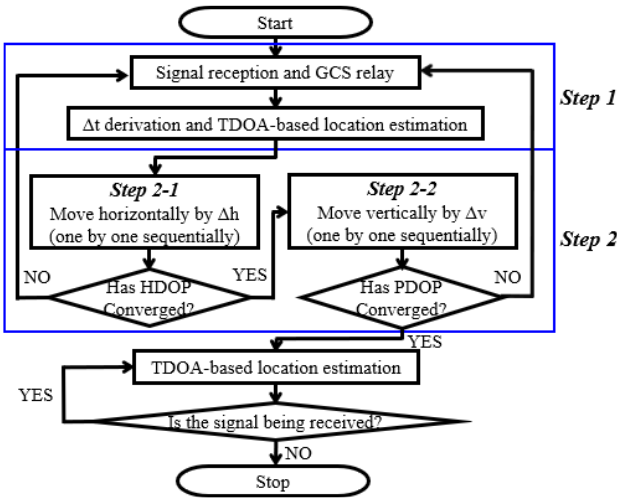

Fig. 2 Flowchart illustrating algorithm of signal source localization

within the movable range R_n at the point where the HDOP is the minimum can be obtained. At this time, when shifting each node, the reference node $\{P_n\}_{n=1}^{n-1}$ is fixed; however, unlike in Step 2-1, vertical movement is performed.

Algorithm 1: *Step 2*

Data: GCS position P_G, n[th] UAVs position P_n, Signal source position P_S, Operation Area radius r, height h, UAV Stepsize Δh, Δv

Result: $\mathrm{HDOP_{min}}$, $\mathrm{PDOP_{min}}$, Estimated location $\widehat{P_S}$

```
Δx = Δh, DOP_Pi = HDOP        // Iteration of Step 2-1
while
  for i=1:n
     DOP_Pi ([P_i, P_i + Δx, P_i − Δx], P_ref, P̂_s)
     find(DOP_min), P_i = P_i ± Δx,
  end
     P̂_s = TDOA(P_n, P_G)          // Estimation of P_s
  if no horizontal movement of all UAVs
     Δx = Δv, DOP_Pi = PDOP // Iteration of Step 2-1
  end
  if no vertical movement of all UAVs, break, end
end
```

Figure 2 shows a flowchart of the proposed algorithm. In Step 1, Δt is derived and the location of the signal source is estimated, and in Step 2, horizontal/vertical movement is performed to estimate the location of the signal source. Algorithm 1 represents the pseudocode for Step 2. In step 2-1, each node is horizontally moved to the position with the minimum HDOP and then the location is estimated. In step 2-2, the node is moved vertically to the position with the minimum PDOP and then the location is estimated.

4. Simulation Results

4.1 Simulation Environments

The operation area is a cylindrical airspace with a limited radius and height [10]. Each point is created by dividing the perimeter of the operational area at a certain altitude by 20/30/60 points, and n UAVs hover over a random point. Each node calculates its position based on the GPS and performs an accurate time synchronization with the GCS. The signal source features surface-to-air surveillance radar specifications [11] and is fixed on the ground at a distance of D_{StG} from the GCS. In one iteration, each UAV calculates the DOP formed by the signal source reference node and each point adjacent to the left, right, upper and lower, and moves horizontally and vertically by $\Delta h/\Delta v$ to the point where the HDOP/PDOP is low. When the movement is completed, the signal source location is estimated. The simulation parameters are presented in Table 1.

Table 1 Simulation parameters

Parameters	Values
Number of Nodes(UAV)	4
Operation Area	r = 15(km), h = 4~6(km)
Step size	Δh = 1.5/3/4.5(km), Δv = 0.1(km)
Distance(D_{StG})	50, 70(km)
Signal Source	RPM = 5(rpm), PRI/PW = 4/0.4(ms)

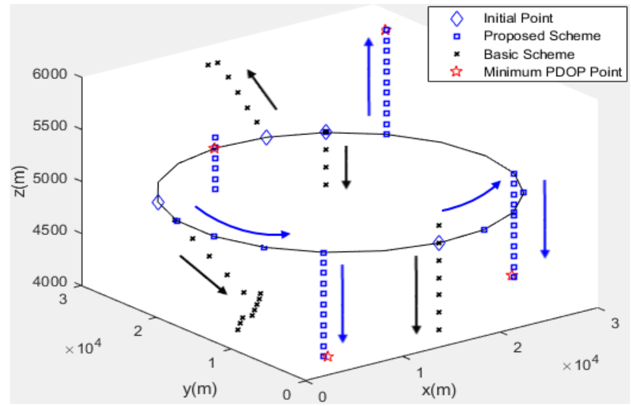

Fig. 3 An example of comparing the trajectories of each node

4.2 Simulation Results

Using MATLAB, the DOP and root mean square error (RMSE) were calculated by repeating the process of 20 times using the proposed algorithm. The PDOP was calculated for all points that each node can move and compared to that obtained using the proposed algorithm.

Figure 3 shows the trajectory of each node. The blue diamond represent the initial positions of each node. The black point represents the trajectory of the basic scheme, where each node shifts after the point with the minimum PDOP is calculated without performing Step 2. The blue point represents the trajectory shifted by performing the proposed algorithm. The red star represents the point that forms the minimum PDOP with the estimated location of the signal source in the operation range. As the iteration proceeded, a location with the minimum PDOP value was successfully identified using the proposed algorithm.

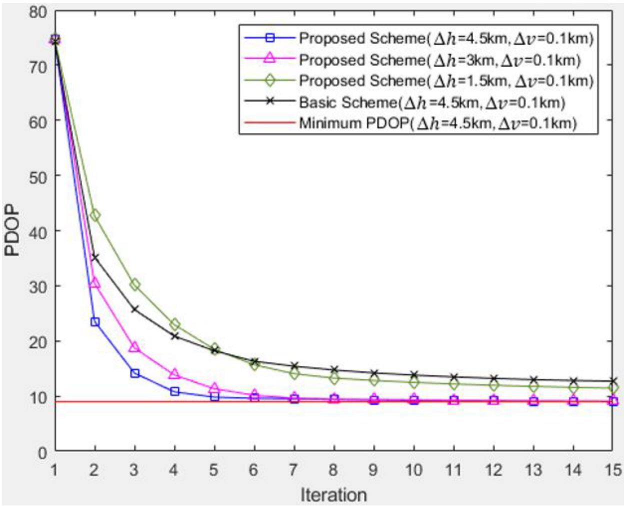

Fig. 4 Comparison of PDOP between basic and proposed schemes based on algorithm iteration

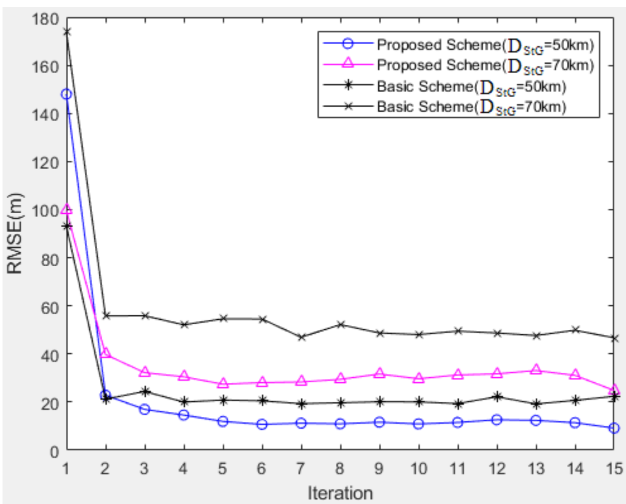

Fig. 5 Comparison of identified location error of basic and proposed schemes based on algorithm iteration

Figure 4 shows a comparison of the PDOP of the node that performed the proposed algorithm and basic scheme. The red line represents the minimum PDOP in the operation area. After performing the proposed algorithm, the node did not remain at the local minimum but approached the global optimum, and the standard deviation converged to 0. In addition, when the step size decreased during one iteration, the number of iterations for convergence increased; however, convergence to the minimum DOP value occurred.

Figure 5 shows a comparison of the RMSE of the proposed algorithm and the basic scheme. The node that shifted through the basic scheme indicated a lower DOP but did not converge to the minimum value; therefore, its RMSE was slightly higher. The results show that the error due to the DOP can be minimized using the proposed algorithm, and that the error can be estimated within approximately 20/40m at a distance of 50/70km from the signal source.

5. Conclusion

To improve the accuracy in estimating the location of unknown radar, we herein proposed an algorithm that allows the optimal position of multiple UAVs to be selected and then shifted, followed by an estimation of the signal source location. In identifying a point that achieves the minimum PDOP in a 3D operation area, it was confirmed that the node shifted to the optimal position via the divide-and-conquer algorithm, which minimized HDOP first and then minimizes PDOP. Thus, the radar location was estimated effectively.

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