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Two-Stage Bicycle Traffic Assignment Model

Seungkyu Ryu[1]; Anthony Chen[2]; Jacqueline Su[3]; and Keechoo Choi[4]

Abstract: Cycling has been considered as a healthy, environmentally friendly, and economical alternative mode of travel to motorized vehicles (especially private motorized vehicles). However, bicycles have often been neglected in the transportation planning and travel demand forecasting modeling processes. The current practice in modeling bicycle trips in a network is either nonexistent or too simplistic. Current practices are simply based on the all-or-nothing (AON) assignment method using single attributes such as distance, safety, or a composite measure of safety multiplied by distance. The purpose of this paper is to develop a two-stage traffic assignment model by considering key factors (or criteria) in cyclist route choice behavior. As an initial effort, the first stage considers two key criteria (distance-related attributes and safety-related attributes) to generate a set of nondominated (or efficient) paths. These two criteria are a composite function of subcriteria. Route distance consists of link distances and intersection turning penalties combined to give the distance-related attribute, while route safety makes use of the bicycle level of service (BLOS) measure developed by the Highway Capacity Manual (HCM) to determine the safety-related attribute. Efficient paths are generated based on the above two key criteria with a biobjective shortest path algorithm. The second stage determines the flow allocation to the set of efficient paths. Several traffic assignment methods are adopted to determine the flow allocations in a network. Numerical experiments are then conducted to demonstrate the two-stage approach for bicycle traffic assignment. Overall, the results of the Winnipeg network demonstrate the applicability of the two-stage bicycle traffic assignment procedure with the flexibility of using different criteria in the first stage to generate efficient paths and different traffic assignment methods in the second stage to allocate flows. **DOI: 10.1061/JTEPBS.0000108.**

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Introduction

Nonmotorized modes such as bicycles constitute an important part of a community's transportation system and are vital to the success of transit-oriented developments (TODs). Yet, they have often been ignored in transportation planning and travel demand forecasting modeling or were at best treated as a by-product in the planning process. Many cities have begun to invest in and to promote cycling as a healthy, environmentally friendly, and economical alternative mode of travel to motorized vehicles (especially private motorized vehicles) (Northrop 2011). However, the current practice in modeling bicycle trips in a network is inadequate, in part because cyclist behavior is not yet fully understood. While auto route choice decisions are governed by a single dominate travel time factor [as given by the Wardrop principle (1952)], cyclist route choice decisions are governed by many influential factors.

Many empirical studies on bicycle route choice analysis indicate that cyclists choose routes based on several criteria (e.g., distance, number of intersections, road grade, bike facility, safety, and so on). Stinson and Bhat (2003), Hunt and Abraham (2007), and Broach et al. (2011) found that cyclists are concerned with travel distance or time when making route choice decisions, while Hopkinson and Wardman (1996), Akar and Clifton (2009), Dill and Carr (2003), Winters et al. (2011), and Lee et al. (2015) indicated that safety played an important role in a cyclist's route decision-making process. Sener et al. (2009) also found that travel distance/time and safety were important factors in cyclists' route choices. Mekuria et al. (2012) suggested that stress is an important factor in bicycle trip-making behavior. Using global positioning system (GPS) tracking data, Hood et al. (2011) developed a path-size logit model (Ben-Akiva and Birelaire 1999) as a cyclist route choice model and performed the bicycle traffic assignment on a pre-enumerated route set generated by the doubly stochastic method (Bovy and Fiorenzo Catalano 2007).

Because of the diverse set of influential factors in bicycle travel, many route planners provide a variety of bicycle routes based on different factors (e.g., least elevation gain route, shortest distance route, safest route, least accident route, bike-friendly route, lowest pollution route, route with green space, etc.) to satisfy the requirements of different cyclists (see Table 1 for a list of selected online bicycle trip planners).

All the routes provided by the online bicycle trip planners are based on a single objective to suit individual cyclists' level of biking experience and on a single dominate criterion affecting the bicycle route choice decision (i.e., shortest route based on distance or safest route based on some measure of safety). These single-criterion routes are not suitable for bicycle traffic assignment because cyclists do not all travel on any one route, but rather on many routes based on different influential factors that can affect cyclist route choice decisions. Currently only a few research efforts focus on network analysis for bicycle trips (e.g., Klobucar and Fricker 2007; Hood et al. 2011; Mekuria et al. 2012). These methods provide an initial effort to develop a traffic assignment method for bicycle trips, but they are too simplistic. They are simply based on the all-or-nothing (AON) assignment method that uses single

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Table 1. Online Bicycle Trip Planners

Route planner	Provided routes
Los Angeles Route Planner	Avoiding elevation gain
	Avoiding pollution
	Preferring green space
	Avoiding prior bicycle accidents
San Francisco Bicycle Trip Planner	Shortest route
	Balanced route
	Bike-friendly route
	Restrictions on gradient
Sacramento Region Bicycle Trip Planner	Shortest route
	Bike-friendly route
Vancouver Cycle Trip Planner	Shortest route
	Least traffic pollution
	Least elevation gain
	Vegetated route
	Restrictions on gradient
Washington D.C. Bike Planner	Shortest route
	Least elevation gain
	Bike-friendly route
New York City Bike Map	Shortest route
	Safe route
	Safer route

attributes such as distance, safety, or a composite measure of safety multiplied by distance.

The purpose of this paper is to develop a two-stage traffic assignment model by considering key factors (or criteria) in cyclist route choice behavior. As an initial effort, the first stage considers two key criteria (distance-related attributes and safety-related attributes) to generate a set of nondominated (or efficient) routes. These two criteria are a composite function of subcriteria [i.e., route distance consists of link distances and intersection delays combined to give the distance-related attribute, while route safety makes use of the Highway Capacity Manual (HCM 2010) bicycle level of service (BLOS) measure, which consists of many subcriteria to determine the safety-related attribute]. Nondominated routes are generated based on the above two key criteria in this first stage with a biobjective shortest path algorithm (e.g., Ehrgott et al. 2012). The second stage determines the flow allocation to the set of nondominated routes. Several traffic assignment methods (i.e., equal share assignment, travel distance per benefit of BLOS assignment, reference point assignment, and dominated area assignment) recently adapted by Raith et al. (2014) from operations research for solving the multiobjective traffic assignment problem are adopted to determine the flow allocations in a bicycle network. In addition, the path-size logit model (i.e., a widely adopted random utility model for discrete choice analysis) is modified as a multipath assignment for the bicycle traffic assignment problem.

This two-stage process is similar to some existing methods (e.g., Hood et al. 2011) that use empirical data for bicycle route generation as a preprocess procedure [i.e., a pre-enumerated route set generated using different route generation methods such as the doubly stochastic method by Bovy and Fiorenzo Catalano (2007) or the breadth-first search link elimination approach by Menghini et al. (2010)] and a standard traffic assignment procedure (i.e., all-or-nothing assignment or multipath assignment) for flow allocation. The two-stage approach adopts a biobjective shortest path problem based on two key attributes to generate nondominated routes and various traffic assignment methods for flow allocation to the nondominated route sets for each origin-destination pair.

The remainder of this paper is organized as follows. After the introduction, the two-stage bicycle traffic assignment procedure is presented, followed by two numerical experiments to demonstrate the features and applicability of the proposed two-stage procedure, and some concluding remarks.

Two-Stage Bicycle Traffic Assignment Procedure

This section describes the proposed two-stage procedure for bicycle traffic assignment as shown in Fig. 1. In Stage 1, two key criteria, namely route distance and route level of service, are used in a biobjective shortest path algorithm to generate a set of nondominated (or efficient) routes. In Stage 2, several traffic assignment methods

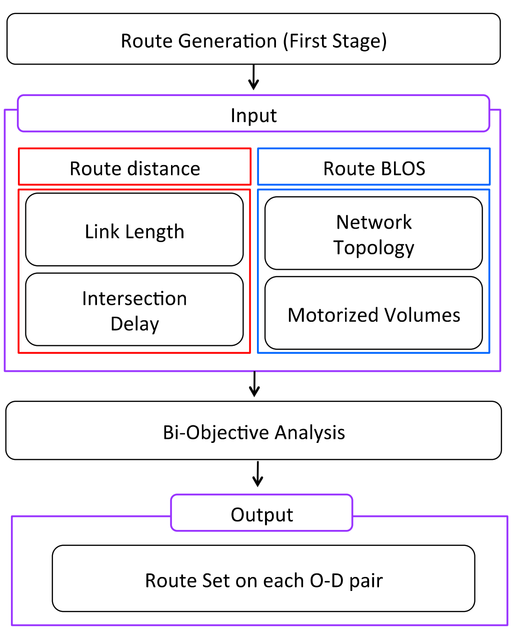

Fig. 1. Two-stage procedure for bicycle traffic assignment

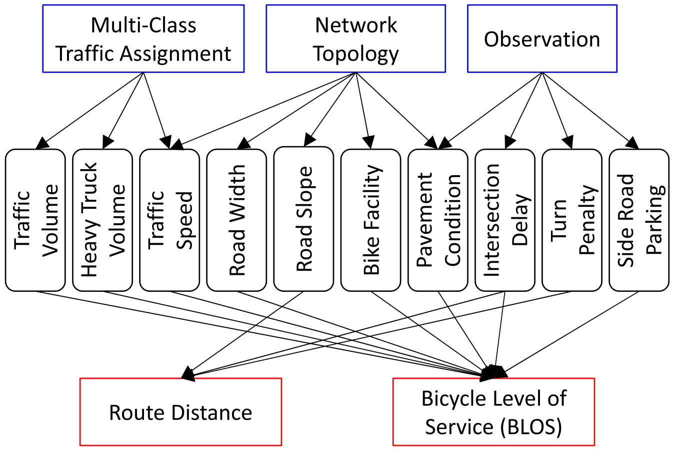

Fig. 2. Two key criteria affecting cyclists' route choice decisions

are adopted to determine the flow allocations to the nondominated routes generated in Stage 1 to obtain the complete bicycle flows on the network. The following subsections describe the two key cyclist route choice criteria, the biobjective shortest path algorithm, and several traffic assignment methods for flow allocations to the nondominated routes.

Two Key Cyclist Route Choice Criteria

Because of the quantity of influential factors in bicycle route choice decisions, using the conventional single objective as the sole criterion for determining route choice decisions as with private motorized vehicles modeling (i.e., the Wardrop user equilibrium model based on flow-dependent travel times) may not be adequate in modeling cyclist route choice behavior (Menghini et al. 2010; Kang and Fricker 2013). From the empirical studies on bicycle route choice reviewed above, two key criteria (distance-related attributes and safety-related attributes) were identified to capture the most important factors affecting cyclist route choice behavior. These two criteria are a composite function of subcriteria as shown in Fig. 2.

Route Distance

Route distance is a composite measure of not only the sum of link distances along the route, but also the turning movement penalties (or delays) at intersections that the route passes through. For bicycle trips, intersection delays have been shown to be a deterrent to cyclist route choice behavior. Since link length and intersection delay measure different qualities (length in meters and time in seconds, respectively), delay is converted to an equivalent distance unit with an appropriate conversion factor. The route distance criterion is computed as follows:

$$d_k^{rs} = \sum_{a \in A} l_a \delta_{ka}^{rs} + \sum_{a \in IN_i} \sum_{b \in OUT_i} cf_i^t d_i^t \delta_{ka}^{rs} \delta_{kb}^{rs}, \quad rs \in \mathrm{RS},\ k \in K_{rs} \tag{1}$$

where d_k^{rs} = distance (in meters) on route k connecting origin-destination (O-D) pair rs; l_a = length (in meters) on link a; δ_{ka}^{rs} (δ_{kb}^{rs}) = route-link indicator; 1 if link a (b) is on route k between O-D pair rs and 0; cf_i^t = penalty conversion factor to equivalent distance unit (in meters/second) for turning movement t at intersection i; d_i^t = penalty (in seconds) of turning movement t at intersection i; A = set of links; IN_i and OUT_i = sets of links terminating into and originating out of intersection i; RS = set of O-D pairs; and K_{rs} = set of routes connecting O-D pair rs. The route distance in Eq. (1) can be computed by summing link distances (first term) and intersection penalties (second term) that make up that route. The first term can further include other attributes such as the penalty for links with elevation gain or restriction on gradient as shown in Table 1, while the second term can include turning movement penalties and/or signalized delays at intersections (i.e., a predetermined value for each turning movement and each intersection, which can be obtained from a traffic signal timing plan or estimated from a traffic assignment procedure with the capability of accounting for turning penalties/intersection delays). Using the intersection turning movement estimation procedure developed by Chen et al. (2012), the turning movement penalty at intersection i is determined by two consecutive route-link indicators $\delta_{ka}^{rs}\delta_{kb}^{rs}$ (i.e., link a and link b along route k between origin r and destination s) without network expansion at each intersection to represent all turning movements. Adding nodes and links to the network to model intersection turning movements is a costly scheme. A standard four-leg intersection would require adding 3 nodes and 12 links to model individual turning movements (left, through, and right) for all approaches. For real networks, it will not only increase the size of the network but also increase the route storage, which will subsequently increase the computation burden of route generation in Stage 1 and flow allocation in Stage 2.

Route Bicycle Level of Service

There are numerous measures for assessing the safety aspect of bicycle facilities or the suitability of infrastructure for bicycle travel. Lowry et al. (2012) provided a recent review of 13 methods used in the literature. All methods attempt to provide a score of the perceived safety of bicycle facilities by using a linear regression with variables that represent conditions of the roadway and the environment that affect a cyclist's comfort level. For this study, the BLOS developed by the Highway Capacity Manual (HCM 2010) is adopted as a surrogate measure to account for different attributes contributing to the safety of bicycle routes. The BLOS measure is considered as the state-of-the-art method and has been adopted by many cities in the United States as a guide for bicycle facility design. However, other bicycle safety measures could also be used in the proposed framework for modeling cyclist route choice behavior. The route BLOS measure described in Eq. (2) is a composite measure based on the average bicycle segment ($ABSeg$) score on a route, average bicycle intersection ($ABInt$) score on a route, and average number of unsignalized conflicts/driveways ($Cflt$) per 1.61 km (1 mi) on a route as follows:

$$\mathrm{BLOS} = 0.200 \cdot (ABSeg) + 0.030 \cdot [\exp(ABInt)] + 0.050 \cdot (Cflt) + 1.40 \tag{2}$$

where l_a = length of link a; $Bseg_a$ = bicycle segment score of link a; $ABSeg = \sum_{a\in k} l_a \cdot Bseg_a / \sum_{a\in k} l_a$ = length weighted average bicycle score on route k; $Int\mathrm{BLOS}_n$ = bicycle score of intersection n; N_k = total number of intersections on route k; and $ABInt = \sum_n Int\mathrm{BLOS}_n / N_k$ = simple intersection average bicycle score on route k.

The segment and intersection bicycle scores ($Bseg_a$ and $Int\mathrm{BLOS}_n$) provided in Eqs. (3) and (4) are calibrated based on the volume and speed of motorized vehicles, width configuration of bicycle facilities, pavement conditions, number of intersections, and so on. The derived BLOS score is a relative measurement without score units to evaluate the level of comfort on the cycling route. The details of the BLOS development can be found in the National Cooperative Highway Research Program (NCHRP) report by Dowling et al. (2008)

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$$BSeg = 0.507 \ln\left(\frac{V}{4 \cdot PHF \cdot L}\right) + 0.199Fs(1 + 10.38 \cdot HV)^2 + 7.066\left(\frac{1}{PC}\right)^2 - 0.005(We)^2 + 0.76 \quad (3)$$

where V = directional motorized vehicle volume given in vehicles/hour (vph); PHF = peak hour factor; L = total number of directional through lanes; Fs = effective speed factor; HV = proportion of heavy vehicles in motorized vehicle volume; PC = Federal Highway Administration's five-point pavement surface condition rating; and We = average effective width of outside through lane given in 0.305 m (1 ft)

$$Int\text{BLOS} = -0.2144 \cdot Wt + 0.0153 \cdot CD + 0.0066\left(\frac{Vol15}{L}\right) + 4.1324 \quad (4)$$

where Wt = width of outside through lane plus paved shoulder (including bike lane where present); CD = crossing distance (the width of the side street including auxiliary lanes and median); and $Vol15$ = volume of directional traffic during a 15-min period.

The calculation of segment and intersection bicycle scores requires not only the volume and speed of motorized vehicles, which are obtained exogenously by solving the multiclass traffic assignment problem with multiple vehicle types, but also detailed network topology information (e.g., pavement surface condition, average effective width of outside through lane, crossing distance, etc.) as shown in Fig. 3. The interaction effect between motorized and nonmotorized vehicles is implicitly accounted for in the BLOS measure, which is used in the first stage for route generation and in the second stage for traffic assignment.

Stage One: Biobjective Shortest Path Procedure

Solving the biobjective shortest path problem is like solving any multiobjective optimization problem because a single optimal solution that dominates all other solutions in all objectives may not exist. Hence, solving multiobjective problems requires generating a set of nondominated (or Pareto) solutions. The biobjective shortest path problem belongs to a class of NP-hard problems (Serafini 1986). Several solution procedures have been developed to solve this complex problem; these include the label correcting approach (Skriver and Andersen 2000), the label setting approach (Tung and Chew 1992), the two-phase method (Ulungu and Teghem 1995), and the ranking method (Climaco and Martins 1982).

Of the two objectives (or criteria) considered for bicycle route generation, the route BLOS measure given in Eq. (2) is not a simple additive sum of the link attributes. Instead, route BLOS is a composite measure based on the average segment bicycle score on a route [$ABSeg$ given in Eq. (3)], the average intersection bicycle score on a route [$ABInt$ given in Eq. (4)], the average number of unsignalized conflicts/driveways per 1.61 km (1 mi) on a route ($Cflt$), and the route-specific constant (1.40). These four terms ($ABSeg$, $ABInt$, $Cflt$, and 1.40) are combined in a nonadditive manner (i.e., not a simple sum of the link/intersection attributes). The handling of nonadditive route cost structure (e.g., route BLOS) may not be easy in the biobjective shortest path problem despite the development of the above solution procedures. In this paper, the ranking method proposed by Climaco and Martins (1982) was modified for solving the multiobjective shortest problem with a nonadditive route cost structure. In the ranking method, no weights are needed since the method explicitly generates a set of nondominated routes. Using a weighted-sum approach, which converts the biobjective (or multiobjective) into a single objective, can only generate one optimal route for a given weight combination. Although multiple routes can be generated by varying the weight combinations, it is well known in the literature that some nondominated routes in the duality gap may not be generated by any weight combinations (Daskin 1995).

The overall modified ranking procedure is described in Fig. 4. In the first step, the procedure uses the distance-related attributes (i.e., link distance and intersection turning movement penalty) to generate a set of realistic routes without exceeding the maximum allowable bound. In the second step, the corresponding safety-related attributes are computed for each route in the set to determine the nondominated routes according to the two key criteria, route distance and route BLOS.

Stage Two: Bicycle Traffic Assignment Methods

Dial (1979) introduced a model and algorithm for the multicriteria route choice problem that aims to proportion travel among routes and modes simultaneously as a traffic assignment model. This model has been extended to a biobjective (or bicriteria) traffic assignment model by adopting a linear value of time (VOT) to convert travel time to an equivalent monetary unit (Dial 1996, 1997). Gabriel and Bernstein (1997), on the other hand, adopted a nonlinear VOT function for the nonadditive traffic equilibrium problem. Nagurney (2000), Nagurney et al. (2001, 2002), and Nagurney and Dong (2002) introduced variable weights for the multicriteria traffic assignment problem by assuming a linear generalized cost function for combining the criteria with variable weights. Recently, Raith et al. (2014) adapted four multiobjective methods from operations research for solving the multiobjective traffic assignment problem. These multiobjective traffic assignment methods have not been

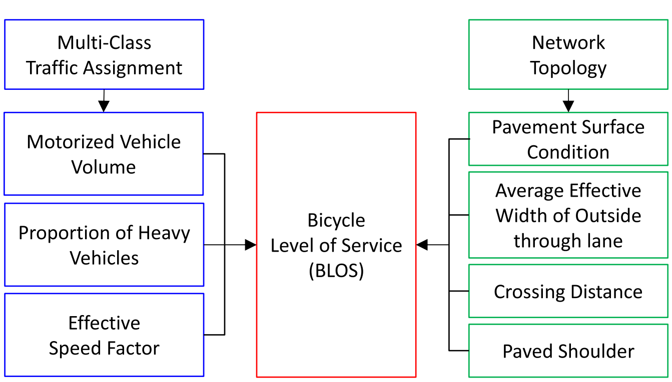

Fig. 3. Input data for computing BLOS

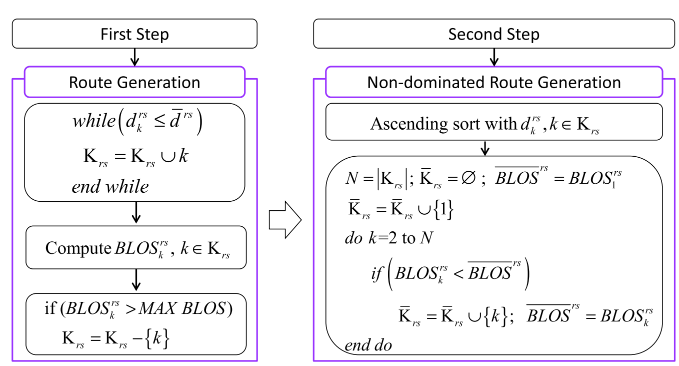

Fig. 4. Modified ranking method for generating nondominated routes

Table 2. Summary of Traffic Assignment Methods

Method	Description	Advantage	Disadvantage	Critical input
Equal share assignment (ESA)	O-D demand is split evenly between all nondominated routes	Easy to implement	Allocated flows not dependent on the objective values	None
Travel distance per benefit of BLOS assignment (TBA)	O-D demand is allocated according to the distribution of travel distance per unit of better BLOS compared to the shortest distance route	Enables the flow allocation to nonsupported routes (nonconvex points) and nonextreme supported routes	Sensitive to the assumed distribution	Distribution of travel distance per benefit of BLOS relative to the shortest distance route
Reference point assignment (RPA)	The route attractiveness is determined by the Euclidean distance to the reference point, and the probability is determined by the route attractiveness relative to the attractiveness of other routes	Easy and intuitive in modifying the shares of demand allocated to each nondominated route	Sensitive to the reference point and potential bias with different objective scales	Reference (ideal) point
Dominated area assignment (DAA)	Shares of demand are allocated to the nondominated routes based on the part of the objective space dominated by the corresponding route attribute point	Considers the attributes of the nondominated routes	Sensitive to the maximum objective values (extreme supported routes)	Maximum value for each objective
Path-size logit assignment (PSLA)	O-D demand is allocated based on the combined utilities of two objectives	Account for the total route cost values and an economic interpretation	Requires detailed survey data to calibrate the parameters	Parameters for the utility function

applied to real transportation networks. In this paper, the authors not only operationalize these methods for solving the bicycle traffic assignment problem, but also compare them to the path-size logit multipath traffic assignment method, a widely adopted random utility model for route choice analysis. Table 2 provides a summary of the traffic assignment methods for flow allocations in Stage 2.

Equal Share Assignment Method

The equal share assignment (ESA) method evenly allocates the O-D demand to all nondominated routes as follows:

$$f_k^{rs} = \frac{q_{rs}}{|K_{rs}|} \tag{5}$$

where f_k^{rs} = flow on route k connecting O-D pair rs; q_{rs} = demand between O-D pair rs; and $|K_{rs}|$ = number of routes in O-D pair rs. Hence, each nondominated route in O-D pair rs has an equal share of the O-D demand.

Travel Distance per Benefit of BLOS Assignment Method

The travel distance per benefit of BLOS assignment (TBA) method allocates the O-D demand according to the distribution of travel distance per benefit of BLOS relative to the shortest distance route. The slopes (ρ) between the shortest distance route and other nondominated routes in the Pareto set represent the travel distance per benefit of BLOS. With the computed slopes, ρ, route choice probabilities can be obtained from a predetermined distribution function as follows:

1. Compute ρ_k^{rs} between the shortest distance route $\bar{k}_{rs}$ and other nondominated routes k of O-D pair rs.
2. Compute route choice probability with ρ_k^{rs}:
   a. $\Pr[\bar{k}_{rs}] = \Pr[0 \le \rho_{\bar{k}}^{rs} < \rho_{\bar{k}+1}^{rs}] = \int_0^{\rho_{\bar{k}+1}^{rs}} f(\rho) d\rho$;
   b. $\Pr[k_{rs}] = \Pr[\rho_k^{rs} \le \rho_k^{rs} < \rho_{k+1}^{rs}] = \int_{\rho_k^{rs}}^{\rho_{k+1}^{rs}} f(\rho) d\rho$;
   c. $\Pr[|K_{rs}|] = 1 - \sum_k^{|K_{rs}|-1} \Pr[k_{rs}]$.

Reference Point Assignment Method

The reference point assignment (RPA) method allocates the O-D demand based on route attractiveness. Route attractiveness is determined by the Euclidean distance (ε) to the reference point (i.e., a virtual or an ideal point), and the route choice probability is determined by the computed route attractiveness. Raith et al. (2014) suggested the following three different probability functions:

$$P_k^{rs} = \frac{\sum_{l=1} \varepsilon_l^{rs} - \varepsilon_k^{rs}}{(|K_{rs}| - 1) \sum_{l=1} \varepsilon_l^{rs}} \tag{6}$$

$$P_k^{rs} = \frac{\sum_{l=1} (\varepsilon_l^{rs})^2 - (\varepsilon_k^{rs})^2}{(|K_{rs}| - 1) \sum_{l=1} (\varepsilon_l^{rs})^2} \tag{7}$$

$$P_k^{rs} = \frac{\prod_{l \neq k} \varepsilon_l^{rs}}{\sum_{l_1=1} \left(\prod_{l_2 \neq l_1} \varepsilon_{l_2}^{rs} \right)} \tag{8}$$

The first function given in Eq. (6) is a sum-based approach. The target ε_k^{rs} is extracted from the sum of all routes $\sum_{l=1} \varepsilon_l^{rs}$, and the probability can be determined by dividing the total Euclidean distance of all routes weighted by the number of routes minus the target route. Alternatively, the probability can be computed based on the square sum as shown in Eq. (7). Finally, the product approach is introduced in Eq. (8).

Dominated Area Assignment Method

The dominated area assignment (DAA) method allocates the O-D demand to the probability obtained from the share space computed with both objective values. See Fig. 5 for an illustration of the share space and route choice probability.

Path-Size Logit Assignment Method

The path-size logit assignment (PSLA) method allocates the O-D demand based on the combined utilities of two objectives via the path-size logit (PSL) choice function. The multinomial logit (MNL) model is a widely used route choice model under the random utility principle. However, it is well known that the major drawback in applying the MNL model to the route choice problem is the inability to account for overlapping (or correlation) among routes. Ben-Akiva and Bierlaire (1999) proposed the PSL model

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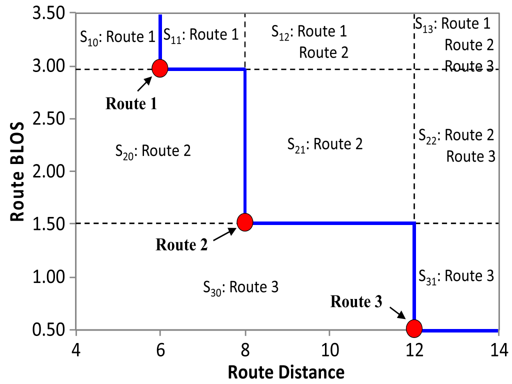


Share space on route k = $\sum_{n=1}^{|K_{rs}|+1-k} S_{kn}/n + S_{k0}$

S: Share space (e.g., S_{21}=6.0)

Hence,

$$P_k^{rs} = \frac{\sum_{n=1}^{|K_{rs}|+1-k} S_{kn}/n + S_{k0}}{\sum_{l=1}^{|K_{rs}|}\sum_{n=1}^{|K_{rs}|+1-l} S_{ln} + \sum_{l=1}^{|K_{rs}|} S_{l0}}$$

Example for route 1

$$P_1^{rs} = \frac{(S_{11} + S_{12}/2 + S_{13}/3 + S_{10})}{30.0} = 0.11$$

Fig. 5. Illustration of route choice probability using the DAA method

as an alternative to solve the route overlapping problem in MNL. The closed-form probability of PSL is expressed as follows:

$$P_k^{rs} = \frac{PS_k^{rs} \cdot \exp(U_k^{rs})}{\sum_{j=1}^{n} PS_j^{rs} \cdot \exp(U_j^{rs})}, \quad \forall\ k \in \mathrm{K}_{rs}, rs \in \mathrm{RS} \tag{9}$$

where $U_k^{rs} = -[(d_k^{rs})^{\alpha} \cdot (\mathrm{BLOS}_k^{rs})^{\beta}]$ = utility of route k between O-D pair rs; $PS_k^{rs} = \sum_{a \in k}\left(\frac{l_a}{L_k^{rs}}\right) \cdot \left(\frac{1}{\sum_{l \in \mathrm{K}_{rs}} \delta_{la}^{rs}}\right)$ = path-size factor of route k between O-D pair rs; L_k^{rs} = length on route k between O-D pair rs; and l_a = length of link a.

Numerical Results

To demonstrate the proposed two-stage bicycle traffic assignment procedure, two networks were adopted in the numerical experiments. First, a simple network was used to illustrate the features of the different traffic assignment methods. Then, a real network was employed to demonstrate the applicability of the two-stage procedure.

Simple Network

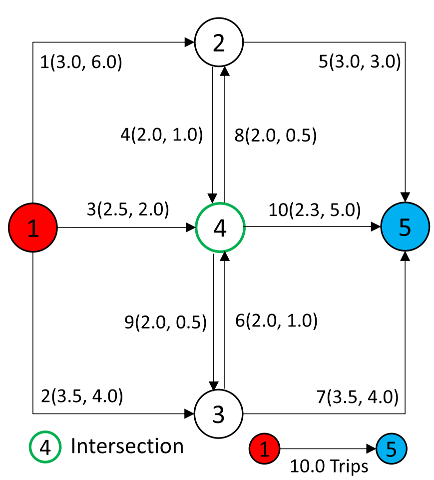
The network shown in Fig. 6 is used to illustrate the features of different traffic assignment methods for bicycle trips. To simplify the analysis, the authors assumed that both objectives (i.e., distance and BLOS) were obtained from a prior analysis. In the left panel, the numbers in parentheses next to each link number are the link distance (in meters) and link BLOS, while the turning delay and intersection BLOS are provided in the right panel. The travel demand from Node 1 to Node 5 is 10 trips.

Using the link characteristics above, the route distance and route BLOS can be computed as shown in Fig. 7. In this experiment, there are five dominated routes (i.e., Routes 2, 3, 7, 8, and 9) and four nondominated routes: Route 1 is the shortest distance route; Route 4 has the lowest BLOS score (a lower BLOS score means a higher level of service); and Routes 5 and 6 are nondominated routes between the two extremes (i.e., they have route distance and route BLOS between the shortest distance route and the least BLOS route).

Comparison of Five Bicycle Traffic Assignment Methods

Using these generated nondominated routes, the following bicycle traffic assignment methods were performed:

- ESA: Uniformly allocate the O-D demand to the four nondominated routes.
- TBA: Gamma distribution with shape $(k) = 2.00$, scale $(\theta) = 2.97$ (i.e., assumed parameters that yield the probability $\Pr[\rho \leq 10.0] = 85\%$).
- RPA: Route distance and route BLOS for the reference point are 5.0 and 1.90, respectively.



(a)

Intersection on node 4		Turning delay	Int. BLOS
From link	To link		
3	8	5.0	1.0
3	10	2.0	1.0
3	9	1.0	1.0
4	10	3.0	1.0
4	9	2.0	2.0
6	8	2.0	2.0
6	10	1.0	2.0

(b)

Fig. 6. Test network and network characteristics: (a) test network and link characteristics; (b) intersection characteristics

Route #	Link member	Route distance (meter)	Route BLOS
1	1-5	6.00	2.33
2	1-4-10	10.30	2.34
3	1-4-9-7	12.50	2.29
4	3-8-5	12.50	1.88
5	3-10	6.80	2.17
6	3-9-7	9.00	1.98
7	2-7	7.00	2.23
8	2-6-10	8.80	2.33
9	2-6-8-5	12.50	2.12

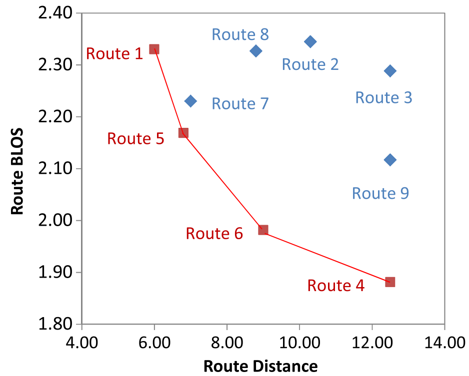


Fig. 7. Estimated route distance and route BLOS and the corresponding generated nondominated routes

- DAA: Maximum distance is 15.0 and maximum route BLOS is 5.0.
- PSLA: Parameters $\alpha = 0.862$, $\beta = 0.117$ of the utility function [obtained from Kang and Fricker (2013)].

Table 3 provides a comparison of allocated flows using the five traffic assignment methods for bicycle trips. From the table, all methods allocate more flows to the shortest distance route (Route 1) except for the ESA and DAA methods. As mentioned, the ESA method allocates an equal amount of flows to all four nondominated routes regardless of the objective values on the routes, while the DAA method allocates flows according to the share space of the route, which is sensitive to the maximum objective values of the extreme supported routes. The TBA and RPA methods allocate flows to the nondominated routes using the objective values (i.e., route distance and route BLOS) in different ways. In the TBA method, the O-D demands are allocated according to the distribution of travel distance per unit of better BLOS compared to the shortest distance route. It enables the flow allocation to nonsupported routes (nonconvex points) and nonextreme supported routes. As for the RPA method, it uses the three probability functions given in Eqs. (6)–(8) to allocate the O-D demand based on route attractiveness determined by the Euclidean distance to the reference point. It is intuitive and easy to modify the shares of demand allocated to each nondominated route. However, both methods do not explicitly consider actual cyclist route choice behavior (i.e., no calibration). The PSLA method, on the other hand, requires additional survey and parameter calibration to fit the cyclists' choice to the two key criteria. In this study, the authors adopt the parameter values from Kang and Fricker (2013). From Table 3, it seems that both TBA and RPA using Eq. (8) can produce allocated flow results like those of the PSLA model. In summary, the ESA and DAA methods, albeit simple, are not suitable for modeling cyclists' route choice behavior since they either do not consider the objective values or they are sensitive to the maximum values when allocating flows to the nondominated routes. The TBA, RPA using Eq. (8), and PSLA methods seem to produce flow patterns that not only account for the objective values but also reflect cyclist route choice behavior.

Table 3. Comparison of Allocated Flows Using Five Assignment Methods

Route number	Route distance	Route BLOS	ESA	TBA	RPA Eq. (6)	RPA Eq. (7)	RPA Eq. (8)	DAA	PSLA
1	6.00	2.33	2.50	4.99	3.08	3.28	4.96	3.78	6.04
5	6.80	2.17	2.50	2.87	2.91	3.19	2.97	0.70	3.06
6	9.00	1.98	2.50	1.70	2.41	2.64	1.35	0.87	0.84
4	12.50	1.88	2.50	0.44	1.60	0.89	0.72	4.64	0.06

Sensitivity Analysis with Different Parameters of the TBA, RPA, and PSLA Methods

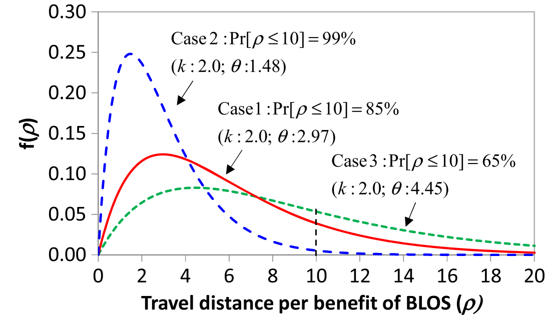
The above analysis indicates that the ESA and DAA methods are inadequate for the biobjective bicycle traffic assignment problem. In the following analyses, several sensitivity tests with different parameters using the TBA, RPA, and PSLA methods were conducted to examine how the parameters affect the route flow allocations. For each assignment method shown in Fig. 8, a figure and a table are used to illustrate the effect of the parameter setting on the assignment method and flow allocation to the nondominated routes, respectively.

- Three cases of the shape and scale parameters of the gamma probability function for TBA (i.e., Case 1: $k = 2.0$ and $\theta = 2.97$; Case 2: $k = 2.0$ and $\theta = 1.48$; Case 3: $k = 2.0$ and $\theta = 4.45$);
- Three cases of the reference point for RPA [i.e., Case 1 = (5.0, 1.9); Case 2 = (5.0, 2.1); Case 3 = (8.0, 1.9)]; and
- Three cases of the two parameters of the utility function for PSLA (i.e., Case 1: $\alpha = 0.862$, $\beta = 0.117$; Case 2: $\alpha = 1.362$, $\beta = 0.117$; Case 3: $\alpha = 0.862$, $\beta = 1.117$) are performed.

As the probability of $\Pr[\rho < 10.0]$ increases in the TBA method (i.e., from 65% in Case 3 to 85% in Case 1 and from 85% in Case 1 to 99% in Case 2), the flow on the shortest route (i.e., Route 1) is significantly increased from 3.07 in Case 3 to 4.99 in Case 1 and from 4.99 in Case 1 to 8.48 in Case 2. Compared to other routes, the flow on Route 1 was more highly affected by the adopted parameter values of the assumed gamma distribution. In the RPA method, the allocated flows are also sensitive to different reference points for calculating route attractiveness. Because route attractiveness is determined by the Euclidean distance to the reference point and because the probability is determined by the route attractiveness relative to the attractiveness of other routes, a nondominated route closer to the reference point would have a higher probability. For the PSLA method, it appears that the distance parameter has a greater effect than the BLOS parameter in the utility function. That is, increasing α from 0.862 in Cases 1 and 3 to 1.362 in Case 2 significantly increases the probability of Route 1 from 0.604 in Case 1 and from 0.577 in Case 3 to 0.914 in Case 2. However, increasing β from 0.117 in Cases 1 and 2 to 1.117 in Case 3 only increases the probability of Route 5 from 0.306 in Case 1 and from 0.086 in Case 2 to 0.355 in Case 3. Overall, all three methods seem to be sensitive to the parameter setting with respect to its assignment method.

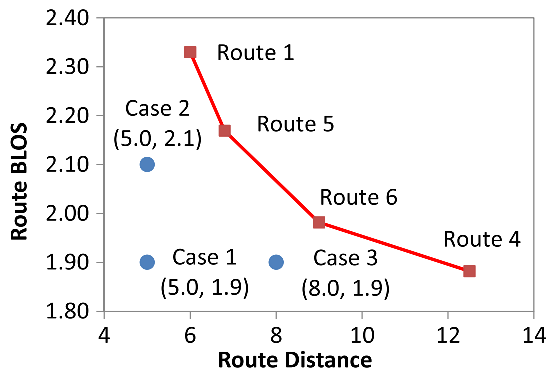
Model	Parameters	Flows

TBA



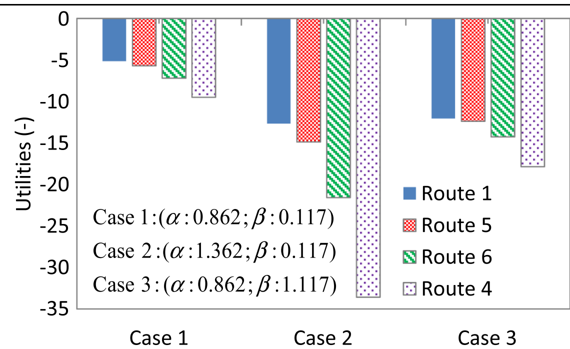
Route #	Case 1	Case 2	Case 3
1	4.99	8.48	3.07
5	2.87	1.32	2.69
6	1.70	0.20	2.60
4	0.44	0.01	1.64

RPA with Eq. (8)



Route #	Case 1	Case 2	Case 3
1	4.96	5.09	1.94
5	2.97	2.90	3.23
6	1.35	1.31	3.95
4	0.72	0.70	0.88

PSLA



Route #	Case 1	Case 2	Case 3
1	6.04	9.14	5.77
5	3.06	0.86	3.55
6	0.84	0.00	0.67
4	0.06	0.00	0.01

Fig. 8. Comparison of allocated flows with different parameters on the TBA, RPA, and PSLA assignment methods

Comparison between Bicriteria and Single-Criterion Assignment Results

In this section, the authors compare the link flow pattern between three bicriteria assignment methods (i.e., TBA, RPA, and PSLA) and two existing single-criterion AON assignments using route distance and route BLOS (i.e., AON-distance and AON-BLOS). The mean absolute error (MAE) and the root-mean-square error (RMSE) were adopted as statistical measures for assessing the link flow differences between each pair of methods. In Fig. 9, RMSE values are shown in the upper triangle, while the MAE values are shown in the lower triangle. The magnitude of the error is indicated by the size of the circle (i.e., a larger circle is associated with a larger error). In general, there is a difference between the single-criterion and bicriteria assignment methods as indicated the larger RMSE (first three columns of the first two rows) and MAE (last three rows of the last two columns) values, implying that the number of criteria used to generate routes and allocate flows is an important factor. Within the three bicriteria assignment methods, RPA and PSLA methods have the most similar link flow pattern as indicated by the lower RMSE and MAE values (i.e., 4.80 and 2.09). As for the two single-criterion AON assignment methods, the link flow patterns are quite different, as indicated by the

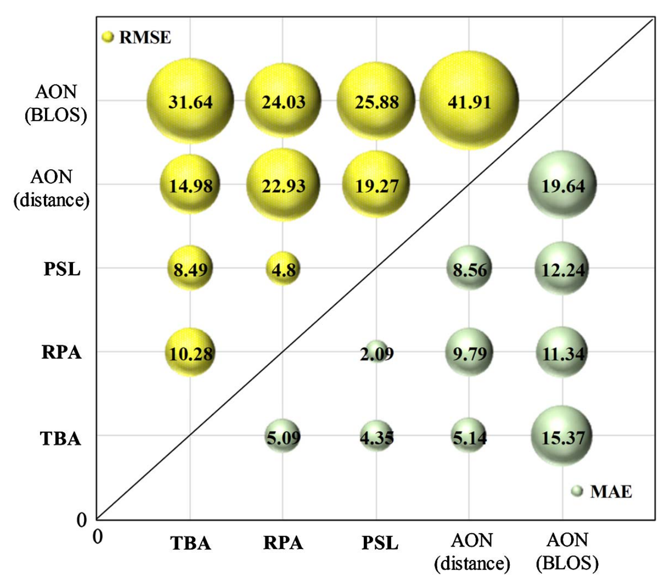


Fig. 9. Link flow comparison between bicriteria (bold text) and single-criterion (unbolded text) assignment methods

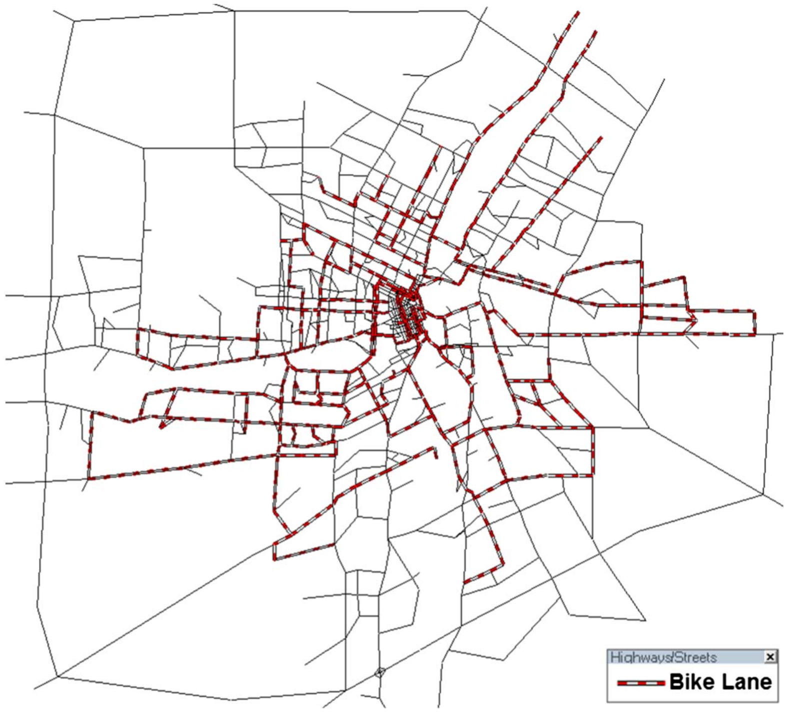


Fig. 10. Winnipeg network with bike lanes

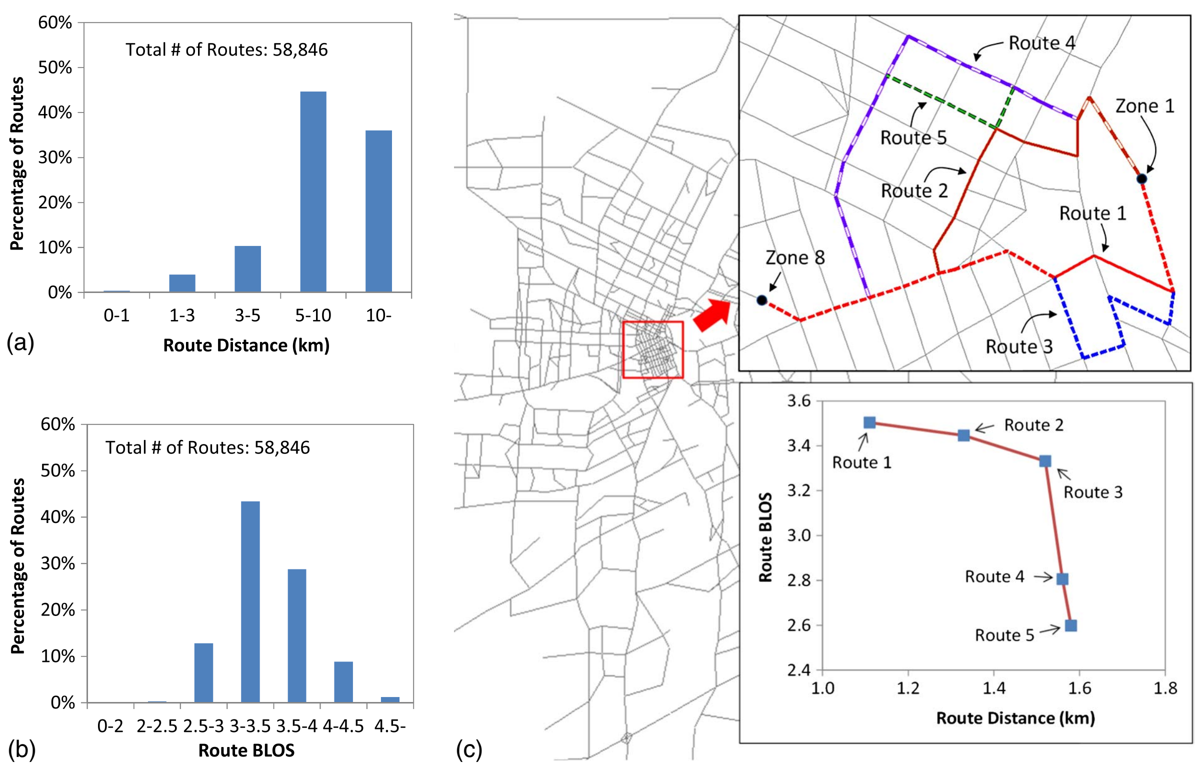


Fig. 11. Generated routes analysis based on route distance and route BLOS: (a) route distribution by distance; (b) route distribution by BLOS; (c) sample routes generated for O-D pair (1-8)

highest RMSE and MAE values (i.e., 41.91 and 19.64), implying that the two criteria (route distance and route BLOS) give quite different routes, which lead to quite different assignment results.

Winnipeg Network

In this section, the two-stage approach is applied to a real network in the city of Winnipeg, Canada. The Winnipeg network, shown in Fig. 10, consists of 154 zones; 1,067 nodes; 2,555 links (1,943 links without centroid connectors); and 4,345 O-D pairs for motorized vehicles. The network structure, O-D trip table for motorized vehicles, and link performance parameters are from the *Emme/4* software. The bicycle network was assembled based on information obtained from the City of Winnipeg (2013b). Among the 2,555 links, 541 links include bike routes or bike lanes. Using the 2006 census data (City of Winnipeg 2013a), the bicycle O-D demand is created based on the gravity model with the gamma function.

The two-stage bicycle traffic assignment procedure was coded in *Intel Visual FORTRAN XE* and runs on a 3.60 GHz processor and 16.00 GB of random access memory (RAM). The total computational effort required was 610–620 s for different assignment problems, about 95% of which is spent in the first stage.

Stage One: Bicycle BLOS Analysis and Route Generation Results

Fig. 11 shows the generated route results in the Winnipeg network. To compute the BLOS measures in Eqs. (3) and (4), traffic conditions (e.g., motorized vehicle volumes) and space availability (e.g., lane width) were obtained from the multiclass traffic assignment results provided by *Emme/4* software and *Google Earth*, respectively. A segment with a high motorized vehicle volume typically gives a higher BLOS value, while links with a larger outside lane width typically give a lower BLOS value. After evaluating the BLOS measures, the modified rank method was performed to generate the nondominated routes in terms of route distance and route BLOS for each O-D pair in the Winnipeg network [Figs. 11(a and b)]. In total, there are 58,846 nondominated routes. Longer distance O-D pairs typically have more nondominated routes, while shorter distance O-D pairs have fewer nondominated routes. As for the route distribution in terms of BLOS, most routes are between 2.5 and 4.0, which correspond to BLOS of B, C, and D. Fig. 11(c) provides an illustration of the nondominated routes for O-D pair (1-8). Route 1 has the shortest distance (1.11 km) and the worst BLOS (3.5), while Route 5 has the best BLOS (2.6) and the longest distance (1.58 km).

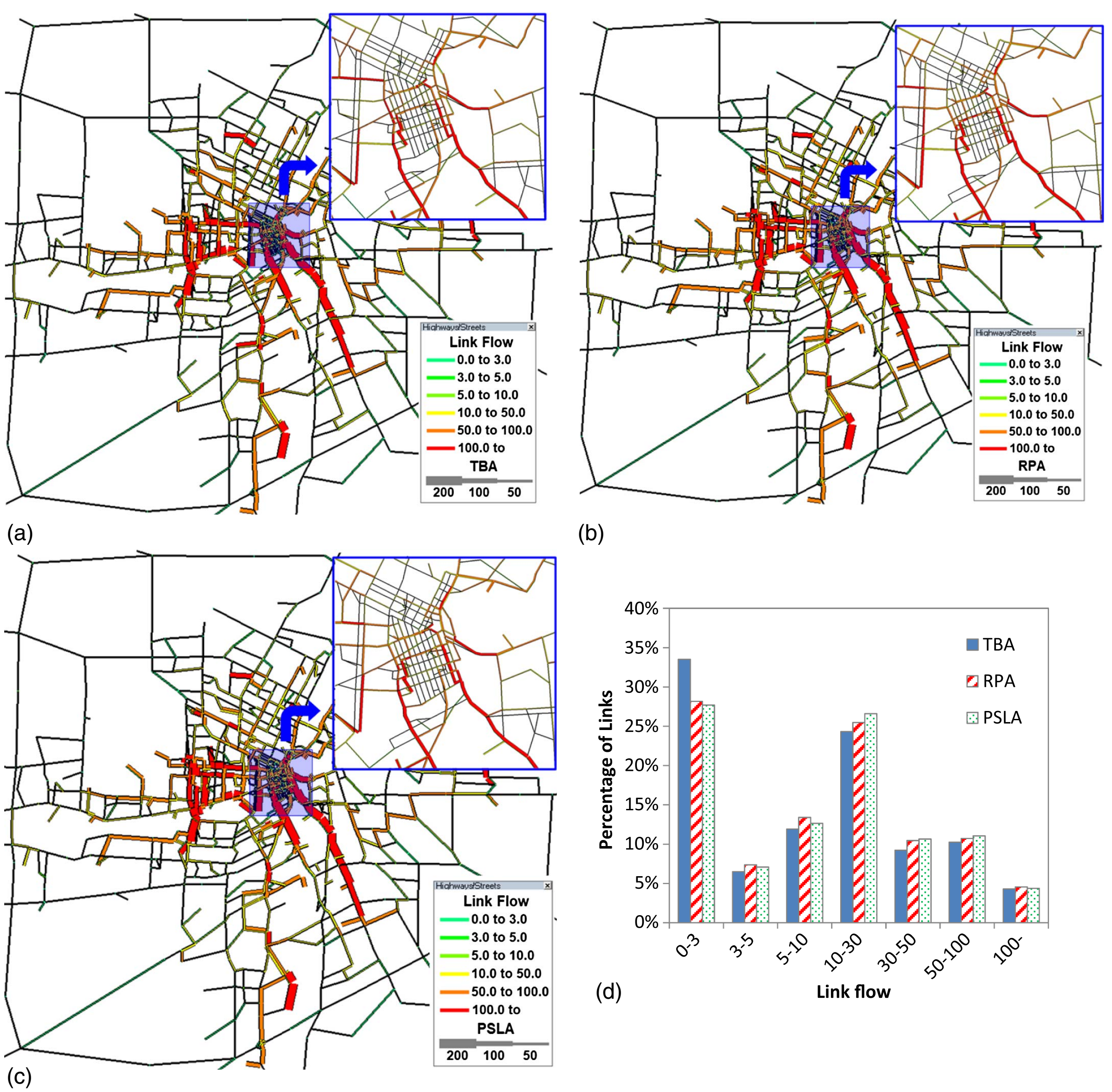
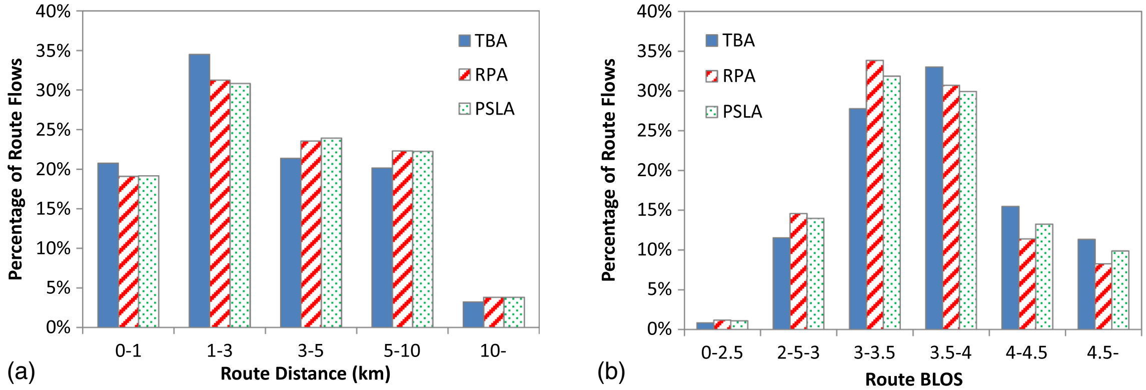


Fig. 12. Link flow patterns of three assignment methods: (a) link flow pattern using TBA; (b) link flow pattern using RPA; (c) link flow pattern using PSLA; (d) link flow distribution



	TBA	RPA	PSLA
TTD	18,607 km	20,142 km	20,161 km
ATD	3.34 km	3.61 km	3.62 km

	TBA	RPA	PSLA
TTB	21,054	20,299	20,626
ATB	3.78	3.64	3.70

Fig. 13. Route flow distribution in terms of route distance and route BLOS: (a) route distance; (b) route BLOS (note: ATB = average traveled BLOS; ATD = average traveled distance; TTB = total traveled BLOS; TTD = total traveled distance)

Stage Two: Bicycle Traffic Assignment Results

Using the generated nondominated routes in the first stage, three bicycle traffic assignment methods were performed: TBA, RPA with Eq. (8), and PSLA with the following assumed parameters:

- TBA: Gamma distribution with $\alpha = 1.50$; $\beta = 0.32$ (i.e., the assumed parameters give the following $\Pr[\rho \leq 1.0\,(\text{km per BLOS})] = 90\%$).
- RPA: Route distance and route BLOS for the reference point are $\min\{d_k^{rs}\}$ and $\min\{BLOS_k^{rs}\}$.
- PSLA: Parameters for the utility function are $\alpha = 0.862$; $\beta = 0.117$ (Kang and Fricker 2013).

Figs. 12(a–c) depict the link flow patterns of TBA, RPA, and PSLA, while Fig. 12(d) compares the link flow distributions of the three assignment methods. Visually, the three link flow patterns look similar.

The main differences from Fig. 12(d) are that TBA and RPA allocate a higher percentage of links to low flow values (i.e., 0–10 units), while PSLA allocates a higher percentage of links to medium flow values (i.e., 10–50 units). For the high flow values (i.e., 50–100+), the three assignment methods identify similar numbers and locations of links in the network as shown by the red color-coded links in Figs. 12(a–c).

In terms of the flow distributions allocated by route distance and route BLOS, Fig. 13 shows the results of the three assignment methods. The TBA method tends to allocate more flows to the shorter distance routes (0–3 km) with a higher value of BLOS or a lower level of safety (3–4.5+), while both the RPA and PSLA methods seem to allocate similar percentages of flows by route distance with some variations by route BLOS. The aggregate measures, total traveled distance (TTD), average traveled distance (ATD), total traveled BLOS (TTB), and average traveled BLOS (ATB), were also computed for the three traffic assignment methods (see the bottom of Fig. 13). Similar to route flow distribution, the TBA method has the lowest TTD and ATD and the highest TTB and ATB. On the other hand, both the RPA and PSLA methods have similar TTD (20,142 and 20,161 km) and ATD (3.61 and 3.62 km), but the RPA method allocates a slightly lower TTB and ATB than those of the PSLA method (20,299 and 20,626 for TTB and 3.64 and 3.70 for ATB). Overall, the results of the Winnipeg network demonstrate the applicability of the two-stage bicycle traffic assignment procedure with the flexibility of using different traffic assignment methods.

Concluding Remarks

In this paper, the authors presented a two-stage bicycle traffic assignment model with consideration of cyclist route choice behavior. In Stage 1, two key criteria (e.g., route distance and route BLOS) were considered to generate a set of nondominated paths using a biobjective shortest path procedure. In Stage 2, five traffic assignment methods (equal share assignment, travel distance per benefit of BLOS assignment, reference point assignment, dominated area assignment, and path-size logit assignment) were adopted for flow allocations to the set of nondominated routes identified in Stage 1.

From the first case study, the authors found that the ESA and DAA methods, albeit simple, are not suitable for modeling cyclists' route choice behavior since these methods either do not consider the objective values or are sensitive to the maximum values when allocating flows to the nondominated routes. The TBA, RPA using Eq. (8), and PSLA methods appeared to produce flow patterns that not only account for the objective values but also reflect cyclists' route choice behavior.

From the second case study, the authors found that there are strong correlations in terms of flow allocations among the TBA, RPA using Eq. (8), and PSLA methods, but that the RPA method seems to allocate a similar flow pattern as the PSLA method. Overall, the results of the Winnipeg network demonstrate the applicability of the two-stage bicycle traffic assignment procedure with the flexibility of using different criteria in the first stage to generate nondominated paths and different traffic assignment methods in the second stage to allocate flows. However, since these results have not been validated with real bicycle data, care should be used when interpreting these assignment results with assumed or borrowed parameters from other studies. The difficulty of validating the model results is the need to have a credible and accurate trip table as an input to the bicycle traffic assignment model. This difficulty is echoed in the work of Hood et al. (2011) using San Francisco as a case study. The validation of the trip assignment results against bicycle counts was poor because of the lack of an accurate bicycle trip table. Hence, it is necessary to develop a bicycle trip table estimation method that can be used in conjunction with a bicycle traffic assignment model.

In this paper, the HCM's bicycle level of service was chosen as a surrogate measure for modeling cyclists' perception of safety (or risk) on different bicycle facility types. It would be helpful to consider other measures, such as the bicycle compatibility index (Harkey et al. 1998) or the stress indicator (Mekuria et al. 2012), and to examine their effect on nondominated route generation and flow allocations to the bicycle network. Additional criteria, such as route pollution linked to health risks (Pankow et al. 2014) and route cognition using the concept of space syntax (Raford et al. 2007), could be considered to model bicycle route choice behavior in the route generation procedure. In addition, more tests should be conducted with different network topologies with different bicycle facilities and travelers' characteristics. The current two-stage bicycle traffic assignment model did not consider the effect of congestion (i.e., link travel times are independent of bicycle flows). As the number of cyclists increases, it would be necessary to consider flow-dependent link travel times to capture the effect of congestion in the two-stage bicycle traffic assignment procedure. Also, multiple user classes should be considered to differentiate different levels of biking experience as well as relevant criteria to reflect different user classes' bicycle route choice behavior. These extensions will further improve the realism of the two-stage bicycle traffic assignment model developed in this paper.

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